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Graphical Representation of Single Elimination Tournament and its Degree Sequences

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Abstract

A single elimination tournament is a type of tournament where the loser of each match is immediately eliminated from the tournament. Each winner will play another match in the nextround. The winner of the final match becomes the single elimination tournament champion. Inthis article, we represent the single elimination tournament as a graph and observe its degree sequences. Also, some results related to the single elimination tournament graph and its win and loss sequences are studied.

Keywords: single elimination tournament, degree sequences, win sequences, loss sequences

AMS Subject Classification: 05C20

1. Introduction

All graphs considered in this article are simple, finite and directed. Unless stated otherwise follow Gary Chartrand and Ping Zhang[2] for graph theory terminology and definitions. The tournament of a graph is studied from[1] . A digraph D is called tournament if for every pair of points *u* and *v* in D there is exactly one arc between *u* and *v*. In Sadiki O. Lewis[4] definedgraphs for Round Robin tournament. He modelled round robin tournaments on tournament graphs which are connected graphs with directed edges. In the Round Robin Tournament, everycompetitor plays with each other exactly once. Here vertices are called teams and edges represent games. Each out-degree represents a win for the particular team. Each in-degree represents a loss for the team. A win sequence $S^+ = (s^+_1, s^+_2, \ldots, s^+_n)$ are the wins of every team on a tournament graph T_G, written in non-increasing order $s^+_1 \geq s^+_2 \geq \ldots \geq s^+_n$. For a vertex

v 1 2 n $i$the number of wins s^{+}=$d^{+}(v_i)$. A lose sequence S^{-} = (s^{-},$^{-}$,…,s^{-}) are the losses of every$i$ 1 2 $n$player on a tournament graph written in increasing order where s^{-} ≤s^{-}≤…≤ s^{-}.For a vertex1 2nv_i the number of losses s^{-}= $d^{-}(v_i)$. Let T_G be a tournament graph on n vertices. We say v is a sink when $d^{+}(v)$ = 0 and hence $d^{-}(v)$ = n-1. Therefore, the degree of v, $d(v)$ = ($d^{+}(v)$,$d^{-}(v)$) = (0, n-1). Similarly, v is a source when $d^{-}(v)$ = 0 and thus $d^{+}(v)$ = n-1. Thus, the degree of v, $d(v)$ = (n-1,0) [4]. In this article, we study the concept of single elimination tournament and it'sdegree sequences.

2.Main Results

The single elimination tournament is introduced by [3]. It is defined as follows,

Definition 2.1. A *single-elimination* (*SE*) *tournament***,** also known as a sudden death tournament, an Olympic tournament, a binary-cup election, is a popular way to select a winneramong multiple candidates/players. In the SE tournament, pairs of players are matchedaccording to an initial seeding, the winners of these pairs advance to the next round, and the losers are eliminated after a single loss. Play continues according to the seeding until a single player, the winner, remains.

Example 2.2. The match fixture of single elimination tournament of 8 (= 2^3) players is

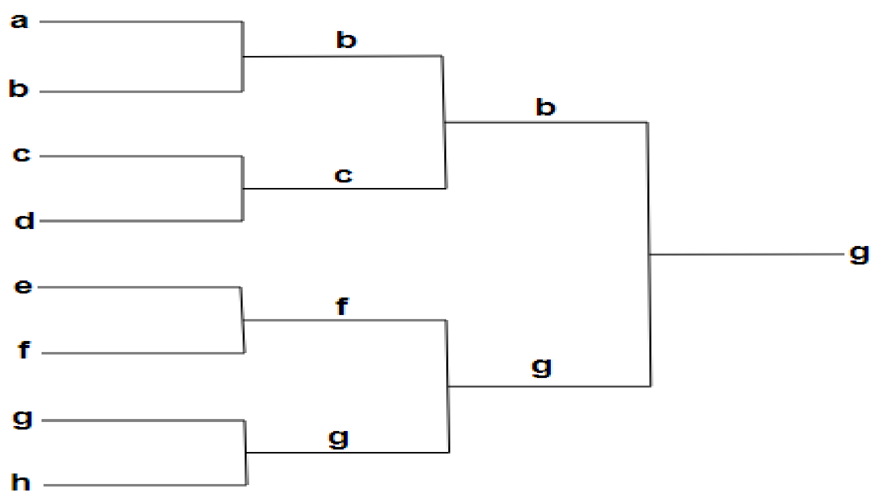

Figure 2.1

Let *a, b, c, d, e, f, g* and *h* be the eight teams playing in this single elimination tournament. In the first round *a* and *b* compete with each other and *b* wins the game, *c* and *d* compete with each other and *c* wins the game, *e* and *f* compete with each other and *f* wins the game and *g* and*h* compete with each other and *g* wins the game. In the second round the winners of the first round compete with each other. *b* and *c* compete with each other and *b* wins the game and *f* and *g* compete with each other and *g* wins the game. In the third round, the winners of the second round *b* and *g* compete with each other and *g* wins the game. *g* is the champion of this single elimination tournament graph. Now by taking teams as vertices and the matches as edges, the winning team as initial vertex and losing team as terminal vertex, the graphical representation of this single elimination tournament is shown in Figure 2.2. The single elimination tournament graph is denoted by G_{ST}.

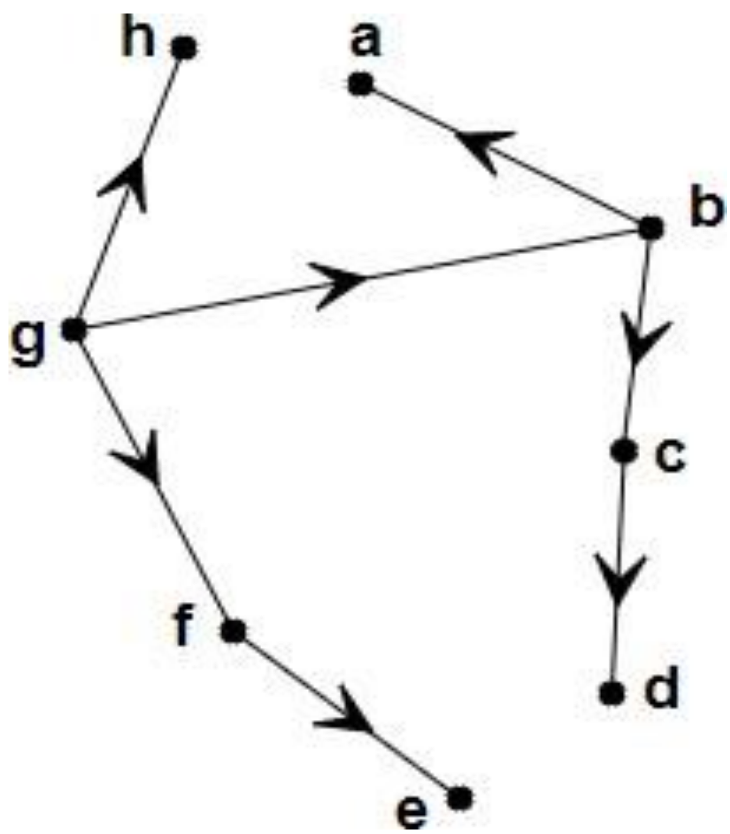

Figure 2.2

Remark 2.3. From the Figure 2.2, we can easily identified that, the win sequence is(3,2,1,1,0,0,0,0) and the lose sequence is (0,1,1,1,1,1,1,1).

Remark 2.4. If the number of teams participating is not a power of 2(irrespective of odd or even), then 'Byes' will be given to a specific number of teams in the first round. The number of 'Byes' to be given is decided by subtracting the number of teams from its next higher numberwhich is the power of 2. The byes of the teams are given the following order:

I Bye - Bottom of the lower halfII Bye - Top of the upper half

III Bye – Top of the bottom half

IV Bye - Bottom of the upper half,…and this procedure continues if the byes to begiven are more than four [5].

Example 2.5. Let *a, b, c, d, e, f, g, h, i, j, k, l, m* and *n* be the 14 teams playing in this singleelimination tournament. The number of teams is 14 which is not a power of 2. So we introduce the term bye here. Number of byes to be given = 2^4-14 = 2.Therefore, in this tournament 2 byes have to be introduced.

The match fixture of this single elimination tournament of 14 players is given below

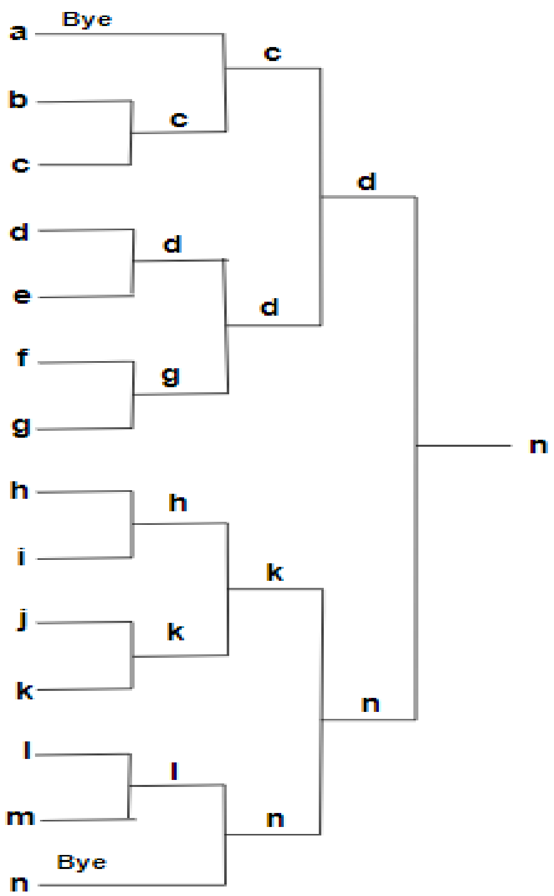

Figure 2.3

In the first round, *a* and *n* are given byes and the other teams compete with each other. *b* and *c* compete with each other and *c* wins the game, *d* and *e* compete with each other and *d* wins the game, *f* and *g* compete with each other and *g* wins the game, *h* and *i* compete with each other and *h* wins the game, *j* and *k* compete with each other and *k* wins the game, *l* and *m* compete with each other and *l* wins the game. In the second round, the winners of the first round and the teams which are given bye in the first round compete with each other. *a* and *c* compete with each other and *c* wins the game, *d* and *g* compete with each other and *d* wins thegame, *h* and *k* compete with each other and *k* wins the game, *l* and *n* compete with each other and *n* wins the game.

In the third round, the winners of the second round compete with each other. *c* and *d* compete with each other and *d* wins the game, *k* and *n* compete with each otherand *n* wins the game. In the fourth round, the winners of the third round *d* and *n* compete witheach other and *n* wins the game. *n* is the champion of this single elimination tournament. Now,by taking teams as vertices and the matches as edges, the winning team as initial vertex and losing team as terminal vertex, the graph G_{ST}, of this single elimination tournament is

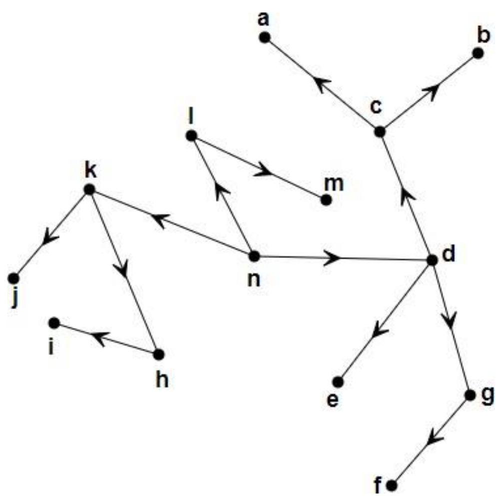

Figure.2.4

Remark 2.6. From the Figure 2.4, it is clear that the win sequenceis (3,3,2,2,1,1,1,0,0,0,0,0,0,0) and the loss sequences (0,1,1,1,1,1,1,1,1,1,1,1,1,1)

Example 2.7. Suppose there are 25 teams playing in this single elimination tournament. Sincethe number of teams is 25 which is not a power of 2, the number of byes to be given = 2^5-25 =

7. Therefore, in this tournament 7 byes have to be given. The procedure for giving byes is givenin Remark 2.4.

Theorem 2.8. A single elimination tournament graph G_{ST}, need not be an orientation of acomplete graph.

Proof. Let G_{ST}, be a single elimination tournament graph with vertex set V and edge set E. Let

$v_{1,2},\ldots,v_n \in V$. Suppose v_1v_2 is a directed edge. Then there is an arrow which contributes outdegree for one vertex and indegree for another vertex. Since, indegree is a loss for a team, the lost team is eliminated from the tournament and further it does not play with any other teams of the tournament. Thus, there exists at least one vertex in G_{ST},which is not connected with every other vertices. Therefore, G_{ST} is not an orientation of a complete graph.

Remark 2.9. The above result does not hold for a single elimination tournament graph with 2vertices.

Observation 2.10. From the above Figure 2.2 and Figure 2.4, we can easily observe that the graphical representation of a single elimination tournament is a tree. Hence, we can say for every single elimination tournament graph G_{ST} contains n-1 edges if it has n vertices.

Theorem 2.11. In a single elimination tournament graph G_{ST}, $\sum s_i^{+} = \sum s_i^{-} = n-1$.

Proof. Consider a single elimination tournament graph with n vertices and n-1 edges. Since there is a win arrow associated with each edge, the sum of all wins equals the total number of arrows in the graph. Since, there is an arrow on each edge, the number of arrows equals the number of edges on the tournament graph. Since, the total number of edges on the graph $G_{ST}$is n-1, we have $\sum s^{+} = n-1$. Similarly, there is a loss arrow associated with each edge and so thesum of the losses equals the number of edges. Hence $\sum s^{-} = n-1$.

In [4] the terminology, Source and Sink are defined for Round Robin Tournament. In this paper, we define Source and Sink for single elimination tournament are as follows.

Definition 2.12. Let G_{ST} be a single elimination tournament graph with n vertices. Let v be a vertex in G_{ST} , then v is a *source* if it's indegree is 0, that is, $d^{-}(v) = 0$.Then the degree of v is$d(v) = (d^+(v), 0)$.

Definition 2.13. Let G_{ST} be a single elimination tournament graph with n vertices. Let v be avertex in G_{ST} , then v is a *sink* if it's outdegree is 0 and indegree is 1, that is $d^+(v) = 0$ and

$d^-(v) = 1$. Then the degree of v is $d(v) = (0, 1)$.

Result 2.14. Let G_{ST} be a single elimination tournament graph with 2^n vertices. Let v be avertex in G_{ST}, then v is a source if it has indegree 0 and outdegree n, that is, $d^-(v) = 0$ and $d^+(v) = n$. Then, the degree of v is $d(v) = (n, 0)$.

Theorem 2.15. A G_{ST} Graph with n vertices has exactly one source.

Proof. In a Single elimination tournament, the matches will take place in rounds. In each round, the team which loss must leave the tournament and the winning team proceeds to the next round. In each round, two teams pair up and compete with each other. By continuing likethis, there will be two teams left in the final round in which one loss and one wins. The team which wins has 0 loss, that is , s^- = 0. Hence the winning team is reffered as a source and thereis no more team with 0 loss. Hence there is exactly one source in G_{ST} graph with n vertices.

Theorem 2.16. Let G_{ST} be a single elimination tournament graph with 2^n vertices. Then it hasexactly 2^{n-1} sinks.

Proof. Consider a single elimination tournament containing 2^n teams. In the first round, therewill be exactly 2^{n-1} teams which loss the match and there will be exactly 2^{n-1} teams which winthe match. The losing teams of the first round has 1 loss and 0 win, that is, $d^+(v) = 0$ and $d^-(v)$
$= 1$. The winning teams of the first round has 1 win, so they cannot be sink. Hence there are exactly 2^{n-1} sinks in G_{ST} graph with 2^n vertices.

Result 2.17. Let G_{ST} be a single elimination tournament graph with n vertices. Then it's loss sequence is of the form $(s^-, s^-, ... , s^-)$ where $s^-= 0$ and $s^-= s^- = = s^- = 1$.

1 2 n 1 2 3 n

Result 2.18. Let G_{ST} be a single elimination tournament graph with 2^n (=p) vertices. Then it's win sequence is of the form $(s_1^+, s_2^+, \ldots, s_p^+)$ where $s_1^+ = n$, $s_2^+ = n-1$, $s_3^+ = s_4^+ = n-2$, $s_5^+ = s_6^+ = s_7^+ = s_8^+ = n-3, \ldots, s^+_{\frac{p}{2}+1} = s^+_{\frac{p}{2}+2} = \ldots = s_p^+ = 0$.